Session Number: **9**

Abstract Name:	**Mathematical Habits of Mind for Teaching**
MSP Project:	Focus on Mathematics, Phase II: Learning Cultures for High Student Achievement
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1. Questions(s) or issue(s) for dialogue at Learning Network Conference session:

What are the mathematical habits of mind that high school teachers use in their professional lives and how can we measure them?

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2. Context of the work within the STEM education literature and within your MSP project:

The notion of mathematical knowledge for teaching has been studied by many researchers, especially at the elementary grades [1,2, 3, 8, 12, 13]. Our understandings of this notion parallel much of what we have read in the literature, but are based on our particular experiences over the past 20 years, as mathematicians engaged in doing mathematics with secondary teachers [19]. As one part of the work of *Focus on Mathematics, Phase II*, we are developing, in collaboration with others in the field, a research program with the ultimate goal of understanding the connections between secondary (grades 7–12) teachers' mathematical knowledge for teaching and secondary students' mathematical understanding and achievement, building on current research in this area [7, 9, 10, 11, 14]. One important feature of our approach is the core involvement of mathematicians in all aspects of the work. By a *research program*, we mean a collection of tightly-knit, large open questions and conjectures that the field deems important. Researchers work on parts of the program—for example, correlating assessment performance with mathematical knowledge used in classrooms, or creating theories about how knowledge is developed or applied. (There are similar research programs in mathematics itself [21], sometimes initiated by individuals and sometimes growing x from a collection of similar research questions and interests. Here too, researchers carve off pieces of the program and obtain partial results, proving general theorems in special cases—over function fields or local fields, for example.) In short, a research program is a dedicated effort with many contributors, many perspectives, and many methodologies, all working towards the ultimate goal of understanding something that is only partially visible, like the top of a mountain that's almost completely under water.

Such a very large and complex program requires the creative efforts of the entire mathematics community. It is especially well suited to collaborations among mathematicians, educators, and teachers. And it is open to all kinds of important research studies—analysis of mathematical knowledge used in high school teaching, the construction and validation of assessments that detect the presence of this knowledge, the effect of content knowledge on classroom practice, the creation of observation protocols that build on the work of Hill [12,13] and others at the elementary level, and, ultimately, studies of effective professional development models that lead to high levels of student achievement. To contribute to this research program, we are in the early

stages of a *focused research study* that will investigate the research question stated in #1 above: *What are the mathematical habits of mind that high school teachers use in their professional lives and how can we measure them?*

Specifically, we will develop two assessment instruments: a paper and pencil assessment of mathematical habits of mind for teaching, and a classroom video observation protocol. These will be modeled on the MKT assessment and MQI protocol developed by Ball and Hill for documenting mathematical knowledge for teaching in elementary teachers [15]. Their assessment measures ``specialized" mathematical knowledge—that is, the knowledge that teachers use, as distinct from the mathematical knowledge held by the general public or used in other professions—whose components include representation of mathematical ideas, careful use of reasoning and explanation, and understanding unique solution approaches. These skills resemble the kinds of mathematical habits that we are interested in studying, at the high school level.

This session will report on progress to date. We will describe our preliminary analysis of mathematical habits of mind for teaching [5,6], share examples of items from the paper and pencil assessments, and report on preliminary observations of about 15 high school classrooms. Both the research program and research study will build on six years of data that has been collected and analyzed by the evaluation team for Focus on Mathematics, Phase I. We will report on this work, focusing on a set of case studies that were conducted with six teachers who have gone through an immersion experience in mathematics [16, 17, 18]. Our evaluators conducted a series of semi-structured interviews with each teacher, interviewed their principals, and made classroom observations of each teacher.

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3. Claim(s) or hypothesis(es) examined in the work (anticipating that veteran projects will have claims, newer projects will have hypotheses):

Rather than claims and conjectures, we have questions. For example, for decades, researchers have claimed that strong teacher content knowledge is a prerequisite for increased student achievement. Over the years, the claim has been refined, both in the nature of the specialized content knowledge used by teachers and in the characterization of genuine student achievement. *Focus on Mathematics* (and before it, *PROMYS for Teachers*) is built on the premise that an immersion experience in mathematics and participation in a mathematical community that involves practicing mathematicians helps teachers develop the habits of mind that are common among mathematicians, and we have some evidence that this is the case. But the question remains: *how do these habits of mind get customized and used in the profession of secondary teaching?* We have some conjectures and even some hopes, but no solid answers. The goal of our Phase II research study is to look more deeply into the effects of ``thinking like a mathematician'' on the work of middle and high school teachers (see [20] for an example of this).

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4. Evaluation and/or research design, data collection and analysis:

The design for the focused research study is to:
(1) develop a detailed definition of mathematical habits of mind for teaching (MHoM-T), building on our prior work on mathematical thinking, the specialized ways of thinking about mathematics (resembling the ways mathematicians think) used by teachers in their professional lives,
(2) develop and pilot test a paper and pencil assessment that measures the nature and degree of mathematical habits of mind for teaching and can be used as a pre and post measure in relation to professional development experiences, including immersion experiences,
(3) develop and pilot a coding scheme for mathematical habits of mind as exhibited in high school level mathematics classroom instruction, and
(4) conduct reliability and validity tests of both the paper and pencil and observation instruments and compare the results to determine the relationship between performance on the paper and pencil test and a focus on mathematical habits of mind in high school mathematics instruction,
5. Key insights (retrospective for veteran projects, prospective for newer projects) that have value for the Learning Network:

The retrospective insights from FoM-I and its predecessors are that experiences in intense mathematical immersion for a sustained period of time, coupled with membership in a permanent mathematical community gives teachers a view of mathematics that's faithful to the nature of the discipline itself. The prospective insight—essentially tied to question #3 above—is that this has implications for teaching practice, and the goal of the next three years is to research the nature of these implications.

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