GETTING STUCK, GETTING UNSTUCK—COACHING AND QUESTIONING

Problem-solving involves being “stuck.” If a task does not puzzle us at all, then it is not a problem; it is an exercise. Sometimes it is appropriate to congratulate a student for being stuck—it means the student has tackled a worthwhile challenge and gotten to a meaningful point.

One way to get stuck is to ask questions that are beyond our background to solve or are not entirely clear. Progress can also halt when we are unable to determine what knowledge would be useful to apply at a particular point. The most common form of “stuck-ness” faced by students stems from their failure to identify the obstacle to their progress. They are stuck because they believe that being stuck is an amorphous and hopeless situation. Teachers and mentors need to help students understand that there are many specific ways to be stuck, and that, for each barrier, there are associated methods for becoming unstuck. The challenge, when stuck, is determining *why* one is stuck.

When students ask questions, the usual response should also be a question. First, you might ask if they can clearly state what they are seeking to determine, or if they can figure out why they are stuck at that stage in the process. Often, students stop in the middle of a task but do not try to characterize what has occurred that has stopped them in their efforts. Encouraging them to identify the cause of their “stuckness” (e.g., “I have too many variables,” “I don’t see any pattern in this sequence”) is frequently all they need in order to focus on, and resolve, their difficulty. As you ask them these questions, also make explicit what you are doing and why: You are asking them the questions you would ask yourself in a comparable position. Encourage them to set up their own internal dialogue in which they continually ask themselves, “What do I know?”, “What do I need to know?”, and “What techniques do I have for bridging the gap?”

Students often become stuck, not because of any impediment, but because they have stopped moving forward. A lack of confidence plays a role in this immobility, and sometimes the needed level of internal dialogue is strikingly simple. Consider the following teacher report:

> On numerous occasions, students have come for extra help because they were stuck with a multi-step problem. After showing me the first step, they would freeze. I ask them, “OK, what do you do next?” They do one more step and, once more, stop. I repeat my question, and they do one more step until the problem is solved.

> They thought I was being helpful, yet I point out that all I did was prompt them to continue. I encourage them to take over that job themselves and remind them that uncertainty should not be allowed to lead to paralysis.

BE A ROLE MODEL

Students will become more comfortable “talking to themselves” if they see the teacher doing likewise. When students ask a question that you cannot answer immediately, try working on it in front of the class. Outline your intuitions about what the answer might be and how a solution might be reached. Try a method and, if it fails, backtrack and start new pathways. This work should be accompanied by the questions you ask yourself each time a next step is encountered (e.g., “Is it time to factor and why?” “Would a graph of this equation help?” “Should I add any segments to this diagram?”). Teachers must be willing to give students these types of apprenticeship experiences. Mathematics students need to see what it looks like to do real mathematics.

Teachers sometimes perceive students as trying to “trip up” the teacher with questions the teacher cannot answer. These students are usually more curious than malicious in their intent and are a font of opportunities for modeling problem-solving. Of course, you may not be able to figure out a solution right away. That’s even better! Take some time at home, share the question with your colleagues or the project mentor, and/or read up on the topic. Demonstrate for the class what persistence and research are really like. If you find such an approach nerve-wracking, then imagine how our students, who are in far less control of their own learning, feel when we subject them to tests and lengthy projects.

TO ANSWER DIRECTLY OR NOT TO ANSWER DIRECTLY

A goal for a research course or strand is for students to learn how to discover new information. Toward that end, teachers need to ask more questions than they answer. However, students need not discover every new fact or skill they need to know. Sometimes, students may be missing one bit of knowledge that would allow them to proceed. In this context, they will really appreciate (and thus be more likely to remember) that information when you explain it to them. In other situations, students may ask questions that they lack the mathematical tools to analyze. In such a case, coaching should involve sparing them a fruitless search. You can direct them toward literature that will enlighten them, or tell them they have asked a deep and nifty question and then teach them about an effective approach to the problem. Helping students have an appreciation for the breadth of the many branches of mathematics may also be facilitated by less Socratic responses.

In contrast, one advantage of question-asking over telling is that we are in a better position to monitor students' learning when they are doing the talking and explaining. If students wonders whether a result or step is correct, ask them how they might check their answer (either by comparing it to the conditions of the problem or by using an alternative method to solve the problem) or ask why they think their result is correct or why they have doubts about it. By not telling them when we think they are right or wrong, they are forced to take checking seriously, to talk to one another, and to make a case for their work. The class becomes a mathematical community instead of a collection of student-teacher dialogues. Lastly, if we are the ultimate source of Truth, then our students will be ill-equipped, once the course ends, to continue using mathematics on their own or with their peers. The effect of this approach comes across clearly in the following student's course evaluation:

> I am more confident about my math now. I'm willing to say what I think is right and back it up with proof, probably because I find it easier to have reasons that back up my thinking. One thing I've learned this year is to be more skeptical of things and to figure them out myself rather than take someone else's word for it. One thing that helped me be more confident was the fact that, no matter if we were right or wrong, you would ask, "Why?" and we had better have a reason. Another thing was that you would never tell us the answer. No matter if the test was over or the problem set handed in, you would still only give us hints so that we could figure it out ourselves. You always believed that we could figure it out ourselves, so we did.

THINK ABOUT IT!

A common feature of difficult problems is that the initial information is ambiguous or incomplete. Problem-solvers often make assumptions about a setting that they do not realize they are making. These assumptions may be unwarranted or may represent one possible interpretation of a problem that is not fully defined. In either case, teachers should help the student think about the information or properties that the student assumed applied. This metacognition is crucial but not simple. The background we bring to a problem is not always explicit in our thinking. Asking students to state what they know and to explain how they know it can be a first step in clarifying their reasoning.

What many students do not realize is that “thinking about something” is an active process. Thinking should involve questioning intuitions, monitoring reasoning, checking definitions and computations, modifying methods and testing cases, planning strategies, comparing results with expectations, or communicating a clear description of the work. For example, information can be represented in different forms. Thinking about a formula might be aided by trying to put the relationship into words. A set of specific numbers might be more revealingly studied through a graph. Thinking requires repeated forays into new representations and transformations of an idea until a useful perspective is discovered. Try to make these activities apparent to the students when they exhibit them and to emphasize that what they just did is what we mean by “Think about it!”

QUESTIONS FOR GETTING UNSTUCK

The first step for getting back on track with a question is to state, more specifically than “I am stuck”, what the nature of the impediment is. Encourage students to continue their sentence: “I am stuck because…” For example, they may realize that they do not know how to find the value for an unknown (in which case, they can refer to various equation-solving techniques). Naming the problem will often suggest a reasonable next step. Any hints offered should be process-oriented (“You seem to need to organize your data in order to look for patterns”). Students may be more comfortable writing about their process or talking through their difficulties. A research log (see Keeping a Research Logbook in the Introduction to Research in the Classroom section) can serve as a diary in which students record their questions and responses.

Here is a list of possible questions students can respond to when stuck:

- **If a research question is not clear:**
  - Can I restate this problem in my own words and symbols? (The introduction of symbols may serve to simplify the statement of the problem and force the student to identify and jettison atmosphere-providing but irrelevant details.)
  - Can I identify the given information and conditions of the problem? What do I know about the (unstated) properties of the objects involved? (For example, let's say you are working with prime numbers, mutually perpendicular vectors, trapezoids, or matrices with a determinant of 1; what "behaviors" distinguish these mathematical objects from similar numbers, vectors, shapes, or matrices?)
  - Does any of the given information seem extraneous? Can I determine whether it truly is?
  - Is there enough information? Have I added, or should I add, any information to the setting?
  - Are there parts of the problem (words or symbols) that I do not understand and need to learn more about?
  - Can I explain what an answer to this problem would look like (a number? method? shape?)?
  - Can I create an example that meets the conditions?
- **If you cannot figure out what mathematical step to apply next:**
  - What different representations can I try in order to clarify the relationships between the elements of my problem?
  - Can I make the situation *visual* by drawing a diagram? (Be cautious. Diagrams can lock you into a single way of looking at a problem. Try to draw several valid diagrams or a general one. Avoid introducing properties [such as parallelism] that are not required in the problem.)
  - Can I make it *abstract* (and also more generalized) by introducing variables?
  - Can I make it *concrete* by finding specific examples (which might be numeric, algebraic, or geometric)?
  - Can I organize the information in some useful way (e.g., in ordered pairs or a table)?

- What information do I need to know in order to proceed? What techniques or theorems connect what I already know with what I need to know?
- Are there any patterns, structures, and/or behaviors that are similar to those in settings I have previously studied?

- **If the problem makes you unhappy:**
  - Does this problem simply lack mathematical appeal for me (e.g., it seems too arbitrary or inelegant)? Sometimes it takes time to be immersed in a problem before its charms become apparent and sometimes a problem simply lacks those charms. These are, of course, quite personal mathematical matters. If choosing a more appealing problem is an option, then consider doing so.
  - Am I put off by the open-ended-ness of the problem? (Encourage the student to start with a specific sub-problem by adding an additional constraint or looking at one particular case.)

USE OF TIME

Leaving students stuck but armed with the above questions is an important stage in developing their persistence and intuitions. Which question is most significant? Which method should be applied? We only gain skill in making such decisions with practice. Good instincts come with experience, and students need to be given the time to gain that experience. Encourage them to work on a problem for a day or two and then take a break if they remain stuck. Our brains are wonderful parallel processors and will often continue mulling over a problem while we attend to other matters, sometimes notifying us (as we do the dishes or take a walk) when a breakthrough has been made.

These “Aha!” moments are wonderful. They will occur when students have the chance to revisit a problem. Let kids lose a little focus on the main goal of a problem as they explore a related digression. Make sure they do return to the main task eventually and see if any new insights have surfaced in the meantime. The occurrence of “Aha!” moments is a good assessment of both teacher and student success in meeting the goals of a research experience.

SOLVE A DIFFERENT QUESTION

If a problem is not yielding any productive results, students should consider solving a related problem (see Creating New Problems from Old Ones in Problem Posing). Ideally, solution of this new problem will inform work on the original question. However, it is not always predictable when ideas will transfer well. The most common and effective related problem is a simpler version of the original (see Simplifying a Problem in Examples, Patterns, and Conjectureshttp://www2.edc.org/makingmath/handbook/teacher/LookingForPatterns/LookingForPatterns.asp).

CHECKING YOUR WORK

Checking one's work is another great way to get stuck. Often, we have a result and are feeling pretty good, but then we check our solution or reconsider our proof and realize that something has gone awry. Students know this phenomenon and work hard to avoid it by assuming flawless thinking on their own part. The end of an effort is a sad time to find out that you have not reached the end after all. One way to help students accept the challenge of producing more robustly reasoned work is to help them incorporate checking as an ongoing, not just after-the-fact, activity. In other words, checking should become inseparable from the broader research process. When students do uncover a false finding, they should once more be earnestly congratulated. The more such habits are highlighted, the more they will be repeated.

Research can be checked in the following ways:

- Finding a different method for reaching the same point (and hoping your results agree)
- Making sure a result really answers the initial question and not some later stage (e.g., students may plug a solution back into an equation and find that it works, without discovering that the equation was not a correct translation of the original problem)
- Explaining how and why each step was undertaken and checking the computational accuracy and the reasoning involved
- Using easier values, conditions, or methods to estimate a range (both demonstrably too high and too low, if possible) that should contain a numerical result, and making sure that the value does indeed land within those boundaries

Adopting a skeptical stance toward one's own work is the most efficient way to learn. Failure to verify conclusions leads to the same mistakes being made repeatedly. When students catch their own mistake and seek to understand the misunderstanding that led to the error, they are much less likely to make the mistake again. If we work hard to point out every logical flaw, students will continue to sit back, relax, and let us do the hard work. Checking is part of the internal dialogue that students must make part of their mathematical habits.

CONCLUSION: BACK TO AFFECT

As a mathematical coach, the teacher has countless difficult decisions to make. Knowing how and when to help our students get stuck and unstuck requires many understandings, most important of which is knowledge of each student and where the student is in her development as a mathematician. If students are to respond productively to being stuck, self-confidence is essential. They need to be able to bring us their anxieties, questions, and ramblings. In response, we have to be cheerleaders, good listeners who can pose good questions, and mentors who can offer opinions about fruitful directions to follow.

This knowledge of our students also affects how we seek and communicate information. One student confessed to a teacher, at the end of months of meetings, that she dreaded the meetings because she knew she would be asked questions that she could not answer. The questions were the right ones for her to consider, but she needed to hear that she was not expected to have instantaneous responses. A written list that she could gradually consult would have conveyed a better message. In this case, the immediacy of the spoken exchange required additional explanation. Throughout the research process, we must be completely open about what each stage should provide for our students. What is appropriate at one point in the process may not be effective if a student does not understand how to respond to new information.

